PAULINE SPERRY, Assistant professor of mathematics, Smith College, Northampton, Mass.
H. IVAH THOMSEN, Baltimore, Md.
ROSS B. WILDERMUTH, Assistant professor of mathematics, Capital University, Columbus, Ohio.
EDNA F. WILSON, De Smet, South Dakota.
JAY W. WOODROW, Assistant professor of physics, University of Colorado, Boulder, Col.
CHIA-CHEOW YEN, Professor of mathematics, Chinese Government Engineering College, Tangshan, China.

To institutional membership:

CULVER-STOCKTON COLLEGE (formerly Christian University), Canton, Mo.
UNIVERSITY OF PORTO RICO, Mayagüez, P. R.

(2) The Council having voted at the New York meeting that the winter meeting of the Association should be held in Chicago in conjunction with the Chicago Section of the American Mathematical Society, the determination of the exact dates for this meeting was referred by the Council to the joint committee on arrangements, the composition of which, as also of the program committee, is to be announced by President Cajori after consultation with the proper authorities of the Society.

(3) Certain contemplated plans which concern the editorial care of the MONTHLY and a proposal to change the manner of choosing the secretary-treasurer are to be communicated to the whole Council and then laid before the members through the columns of the MONTHLY, in accordance with the provisions of the constitution concerning amendments to the constitution or by-laws.

(4) Measures for taking care of certain expense in the collecting of data for the National Committee on Mathematical Requirements were referred with power to the Committee on Finance of the Council.

(5) A plan for a mathematical dictionary, suggested in the main by Professor G. A. Miller, was brought by him before the Council. It was agreed that the plan, if it can be carried out, is so important and that the questions of the various details, as to size, cost, extent, etc., are so involved, that a committee of five should be appointed by the president, to make an extended study of the plan, and to report to the Council.

W. D. CAIRNS, *Secretary-Treasurer.*

ALGEBRA COURSES FOR COLLEGE JUNIORS AND SENIORS.

Edited by U. G. MITCHELL, University of Kansas.

At the fourth meeting of the Kansas Section of the Mathematical Association of America a large part of the program was devoted to the consideration of

algebra courses for college students. Students taking such courses are, for the most part, preparing to teach in secondary schools, to do research work in mathematics, or to enter applied sciences. The plan of the program committee was to have three men present the subject independently from these three different points of view and by noting the material common to the three selections proposed determine the basis for a course which would answer for all three classes of students. Summaries of the papers in the order of their presentation and of the conclusions brought out in the general discussion are given below.

A. FOR STUDENTS PREPARING TO TEACH, BY U. G. MITCHELL, University of Kansas.

We take for granted that the college juniors and seniors referred to have taken, during the first two years of their college life, the usual freshman and sophomore courses in college algebra, plane trigonometry, analytical geometry and calculus.

The chief consideration in determining such junior-senior algebra courses is unquestionably the prospective teacher's needs which a study of algebra can supply. Laying aside, for the moment, the question as to what needs can be supplied by algebra, some of the teacher's needs which come immediately to mind are:

1. *Interpreting power.* The teacher needs to grasp quickly and accurately the meaning of a problem, whether abstract or concrete, whether stated orally or in print, and to translate this meaning into mathematical terms. This power depends largely, it is true, upon innate mentality; but it also depends largely upon familiarity with forms of statement and with the concepts and conditions involved. It is a power which may be greatly cultivated, even in persons whose mental processes are not naturally quick.

2. *Analytical Power.* By this term is meant the power to separate conditions or variables from each other and to comprehend readily their interrelations. This implies, of course, synthesizing power as well.

3. *Mathematical perspective.* The teacher needs mathematical knowledge which goes much beyond that which he has to teach in order to estimate properly the relative importance of what he teaches. For example, the teacher who has solved no equations of higher degree than the second and the teacher who is familiar with the solutions of the cubic and biquadratic and knows the essentials of the theory of elimination and the Galois theory of equations will view the quadratic equation in very different perspectives.

4. *Facility in performing algebraic operations.* Facility is here used to imply not only speed but also such mastery as combines ease of performance with accuracy.

5. *An understanding of the number system as a logical development.* The number system is first learned as a practical and not as a logical system. There is no attempt to formulate explicitly the assumptions underlying algebra in any way comparable to the recognition of the definitions, axioms and postulates of elementary geometry. Consequently students have no precise knowledge of

these assumptions or of their limitations. They readily extend to negative and imaginary numbers assumptions applicable to positive numbers only. It will be an unusually bright junior class in which there is unanimity of opinion as to the product of $\sqrt{-3}$ by $\sqrt{-5}$ or in locating the error in such simple fallacies as the "proofs" [1] that $2 = 1$ and $-1 = 1$. The teacher of elementary algebra should certainly know enough concerning the assumptions underlying algebra to be sure of his ground within the scope of his teaching. It is only by such a knowledge that he gains sufficient mastery to know definitely and exactly what operations, processes, and manipulations of symbols are valid and what are not permissible.

6. *An understanding of elementary mathematics as a historical development.* Biologists tell us that, in broad outline, the child in his development repeats the experience of his ancestry—that the theory of recapitulation obtains. Whether or not this be true in general, it is certainly true that the student who has studied successfully the historical development of mathematics gets an objective view of his own mathematical evolution, much as the study of Latin or other foreign language gives the student an objective view of English, which can be obtained in no other way. The teacher needs this view in order to understand and direct most intelligently the mathematical development of those under his instruction. It also proves an excellent source of information from which enriching and stimulating material can be drawn.

Other needs, possibly as important as some of these, might be added; but considering only these, for our present purposes, we turn to the question of what material of an algebraic nature can be used to supply these needs.

For developing *interpreting power,* problems exhibiting functional relations from any other sciences or life-conditions, graphs involving such functional relations, and problems involving pure number relations would seem to be the most suitable material. *Analytical power* can also be developed by such problems as well as by algebraic proofs.

All algebraic material would, of course, contribute to *mathematical perspective;* but the following may be selected as best adapted for the purpose: sequences, limits, theorems on limits, series (including convergence tests, exponential, binomial and logarithmic series and operations with series), permutations and combinations, multinomial theorem, symmetric functions of roots of equations, transformations of equations, solutions of cubic and biquadratic equations, systems of linear equations, determinants and elimination, simultaneous quadratics, methods of approximating roots of numerical equations (Newton's and Horner's), fundamental theorem of algebra and its corollaries.

Facility in performing algebraic operations will be aided by the work in theory of equations and can be obtained by the use of exercises of sufficient difficulty; but the probability is that juniors and seniors will have gained considerable (possibly adequate) facility of this kind from the large amount of algebraic work in the differential and integral calculus.

[1] Cf. BALL'S *Mathematical Recreations and Essays,* fourth ed., pp. 24–26.

To gain an *understanding of the number system as a logical development* the student may begin with definitions of cardinal and ordinal integers (positive, of course) and formulate exactly the laws of order and procedure for that system. The number system is next enlarged by introducing fractions on an ordinal basis and reconsidering the laws of procedure and order in relation to these new numbers. Similarly, the system is successively enlarged by the introduction of negative, irrational and complex numbers, taking care at the time of each enlargment to make such extensions, restatements and modifications of the previous laws as will apply. Approximately such a development is found, for example, in the first 75 pages of Fine's *College Algebra.*

There is, of course, no strictly algebraic material which can give an *understanding of the number-system as a historical development.* Such an understanding is best obtained from a course in the history of mathematics. If, however, no such course is given to juniors and seniors, it would seem as if some such historical material should be studied in connection with the study of the logical development of the number system.

Sufficient material has been suggested for a three-hour course throughout a year. The first semester's work might well consist of the theory of equations and related topics and the second semester's work of the logical (and somewhat historical) development of the number system, sequences, limits and series.

B. For Students Preparing to Do Research Work, by W. H. Garrett, Baker University.

It is evident that the junior-senior course in algebra will depend as to its character and extent on the course in algebra which has preceded it—the "college algebra" course which the student has taken as a freshman. I shall suppose that the junior or senior has had such a course, in which the usual topics in theory of equations and determinants have been treated in a brief and elementary way. I believe the tendency of the present day is to cut the course in algebra to the smallest dimensions possible, reserving the discussion of the more abstract parts until after the student has had courses in analytic geometry and calculus. In this connection I may say that at Harvard a student may take Mathematics 3—Introduction to Modern Algebra and Geometry—without having had a course in college algebra and so, in that course, it is necessary to take up the study of determinants from the very beginning. It is my opinion, however, that the average student will do well to have taken the college algebra before entering such a course.

Supposing, then, that the student has had a brief course in college algebra, the question remains as to what topics to present in the junior-senior course. Here, again, I find the tendency is to make the course as brief as possible. In fact, in two of the large universities, there is no mention in their catalogs of such a course being offered to undergraduates. On the other hand, more time and attention is being given to courses along geometrical lines which seem to many to be more interesting and important at that stage of the student's progress. A

three-hour course for one semester should afford ample time to cover all the necessary topics in algebra the undergraduate needs as a preparation for graduate courses and with some classes the ground could be adequately covered in two lectures a week for one semester.

In such a three-hour course there would be time for the presentation of most, if not all, of the topics named below, although not necessarily in the order mentioned. For example, there appears to be no general agreement as to whether the theory of determinants or equations should be taken up first.

Two of the topics would be complex numbers with their geometric representation and the proof of DeMoivre's theorem by mathematical induction. The proof by mathematical induction often given in the college algebra course, would better be postponed, I believe, until this time.

Under the theory of equations I should recommend about such topics and treatment as are given in Dickson's *Elementary Theory of Equations*, including the general properties of polynomials, their continuity and graphical representation, a clear statement of the fundamental theorem of algebra and the necessity for a proof, corollaries following the fundamental theorem, relation of roots to coefficients, imaginary roots, Descartes's rule of signs, elementary transformations of equations, the solution of equations by Newton's method rather than by Horner's, the algebraic solution of the cubic and quartic, multiple roots and Sturm's theorem.

I would not include the proof of the fundamental theorem of algebra. I believe it is time wasted to try to present its proof to the average junior-senior class, since any proof that can be given at this stage will seem long and difficult to the student and will not really teach him anything; whereas, if the proof is postponed to a first course in the theory of functions, it can be made very simple and instructive. In connection with the discussion of Sturm's theorem there may be need for teaching the method of finding the highest common divisor by continued division, a method which was formerly taught in every course in elementary algebra, but which is now generally relegated to the appendix or eliminated altogether.

Under optional topics might be included reciprocal equations, the elementary theory of symmetric functions and its applications to resultants and discriminants of polynomials in a single variable.

In connection with the study of determinants there is a good opportunity to do some constructive work which will be of direct use to the student later on in his graduate work. Following the definition of a determinant and some of the elementary theorems, including Laplace's development and its application to the multiplication theorem, there should be emphasized the topic of linear dependence and the application of determinants to the solution of linear equations, including the solution of systems in which the number of unknowns is not equal to the number of equations, and including systems of homogeneous and non-homogeneous equations. In this connection the first four or five chapters in Bôcher's *Introduction to Higher Algebra* present just what is needed, although the

method would probably have to be modified. The notion of rank of a determinant should be introduced as early as possible and its importance emphasized. In addition, if there is time, some work could be done in elementary invariants.

It may be said that many of the foregoing topics are valuable and interesting for their own sake, rather than as a means of preparation for higher courses in the subject, but I believe that there is need of a certain amount of such work to serve as a transition from the elementary concrete algebra of the freshman year to the abstract and highly developed algebra with its formidable notation and involved formulas which is given in the graduate school.

It is gratifying to learn from some university professors who have in their classes graduate students fresh from their college courses in mathematics, that the average first-year graduate student is not poorly prepared in algebra, but that rather it seems that this is a subject in which he is pretty well prepared and that there is no complaint to be made on this score. Perhaps not so much can be said concerning the student's preparation in geometry, and, in some colleges at least, the emphasis needs to be laid on the geometry course for juniors and seniors rather than on the algebra course.

C. For Students Preparing to Enter Applied Sciences, by A. R. Crathorne, University of Illinois.

If a student in applied science has time to take a course in algebra after the regulation courses in analytic geometry and calculus the course which would be of most benefit to him would emphasize the solution of equations, theory of probabilities, determinants and infinite series. The topics in such a course most helpful to the technical student are: General properties of equations, Descartes's rule of signs, relations between roots and coefficients, transformations of equations, algebraic solution of cubic and biquadratic equations,[1] equations whose degree exceeds four, derived functions, Rolle's theorem, separation of roots; equations in one unknown and with numerical coefficients—multiple roots, Sturm's theorem, rational roots, irrational roots, solutions by means of graphs and machines, comparison of Newton's and Horner's methods, trigonometric solutions; simultaneous equations—consistency, consistency of two equations in one unknown, equivalent equations; transcendental equations, permutations and combinations leading to the theory of probabilities and least squares, determinants, including elimination, and infinite series.

However, most students of applied science do not find a place in their course for much mathematics beyond the calculus. If they can take but one course, I think that a course in advanced calculus and differential equations emphasizing the following topics would be most useful: Differential equations, emphasis being placed on the derivation of equations, on graphical and approximate solutions and on solutions in series; definite integrals, including differentiation with respect to a parameter in the integrand or in the limits of integration; expansion

[1] These solutions of the cubic and biquadratic are not particularly useful to a technical student, but he often loses much time in finding this out. He often expects a simple solution such as he has found for the quadratic equation.

in series other than power series; partial differentiation, especially of composite functions. [For example, a clear understanding of the expression

$$\frac{du}{dx}=\frac{\partial u}{\partial x}+\frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x},$$

where $u = f(x, y, z)$ and $z = \varphi(x, y)$.] Line integrals, with Stokes's theorem, vector analysis.

D. Conclusions.

During the general discussion the speakers modified slightly their choices of material. The following was practically agreed upon as the basis for a course suitable for all three classes of students and follows very nearly the material common to the three original selections. The order here given is not to be considered as indicating necessarily the best order of presentation: Systems of linear equations, determinants and elimination, graphical representation of polynomials, transformations of equations, solutions of the cubic and biquadratic, methods of locating and approximating roots of numerical equations—Descartes's rule of signs, Sturm's theorem, multiple roots, Horner's method of approximation, Newton's method of approximation (if only one method of approximation is given, Newton's is to be preferred), fundamental theorem of algebra and its corollaries, symmetric functions of the roots of an equation, permutations and combinations, problems involving pure number-relations.

The subject-matter above suggested would furnish about enough material for a three-hour course during one semester.

A LIST OF MATHEMATICAL BOOKS FOR SCHOOLS AND COLLEGES.[1]

Preliminary Statement. The following list contains the titles of 160 books which it is believed are suitable for purchase by the usual school or college library.[2] The plan has been carried out of forming the books into four divisions of 40 each, corresponding roughly to the mathematical attainments of the freshman, sophomore, junior and senior years, this arrangement also indicating incidentally the relative difficulty which the books present. Preference has been given throughout to books in the English language, and no text-books have been included except a few having a well-recognized value as books of reference. As it might be desirable for a school or college to know in advance the price of a

[1] Reference may be made to a similar list, but intended more especially for high schools and normal schools, which has recently been published under the auspices of the Teachers College, Columbia University, New York. The title is "A brief list of mathematical books suitable for libraries in high schools and normal schools," *Teachers College Bulletin*, Series 8, No. 3 (1916).

[2] The Library Committee was appointed at the Cambridge meeting of the Association in September, 1916, and a preliminary report was made at the New York meeting in December, 1916. (See the February Monthly, page 58.) Of the numerous tentative plans mentioned in this report, the following pages give the results of several months' work of the committee on the one considered of first importance. Editors.